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moving to L and G falling on M, and generating $CEALE$, and through MI a plane be passed parallel to CEA, the semi-segment $CGK=EFS$ generates $CKILMG$, of which the part $INLM=EFHS$.

The volume generated by the semi-segment EFS in an entire revolution will equal that generated by CGK *minus* the sum of the solids $CKNIGM$ lying about the circumference of the base of the hemi-sphere. But $GMCKIN=KI\times$ area of the semi-segment CGK; and the sum of all these parts is equal to the circumference described by EK as radius into the same area. If $EA=a$, $EI=c$, $ES=s$, and the arc $SF=p$, we obtain for the solid generated by $EAGK$, $\frac{2\pi}{3}(sc^2+a^3-a^2s)$. Consequently the solid generated by $CGK=\frac{2\pi}{3}(a^2s-sc^2)$.

Then the sum of all the solids $CNIKMG=$semi-segment $GCK\times 2\pi c=\pi(cap-sc^2)$, and the volume sought is $\frac{2\pi}{3}(a^2s-sc^2)-\pi(cap-sc^2)=\pi(\frac{sc^2}{3}+\frac{2a^2s}{3}-cap)$. Putting $c^2=a^2-s^2$ this becomes, $\pi(sa^2-\frac{s^3}{3}-cap)$. In the problem $s=\frac{a}{2}\sqrt{3}$, $c=\frac{a}{2}$, $p=a(60^\circ)=\frac{a\pi}{3}$. Then $cap=\frac{a^3\pi}{2.3}$, and the contents are $\frac{a^3\pi}{2}\left(\frac{3\sqrt{3}}{4}-\frac{\pi}{3}\right)=395.59$ cubic feet. This method of solution was suggested by a solution of a similar problem by Professor Seyford, of Colby University.

[From the MONTHLY of October, 1894, pp. 257-8, we have *four* other different solutions of a similar problem; and *each* solution gives the result, $V=\frac{1}{3}\pi(9\sqrt{3}-4\pi)a^3$, $\frac{125}{3}\pi(9\sqrt{3}-4\pi)=395.59027+$ cubic feet. EDITOR.]

39. Proposed by J. C. GREGG, Brazil, Indiana.

Show that the curve

$$x=9a\sin\theta-4a\sin^3\theta$$
$$y=-3a\cos\theta+4a\cos^3\theta$$

is symmetrical to the axes, and has double points and cusps: find the lengths of the arcs, (a) between the double points, (b) between a double point and a cusp, (c) and the arc connecting two cusps and not passing through a double point. [*Johnson's Calculus.*]

Solution by GEORGE B. McCLELLAN ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

The equations as given in *Johnson's Calculus* are

$x=9a\sin\theta-4a\sin^3\theta=6a\sin\theta+a\sin3\theta$,

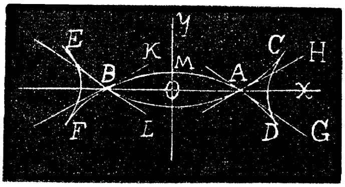
$y=-3a\cos\theta+4a\cos^3\theta=a\cos3\theta$.

$\therefore r^2=x^2+y^2=a^2+72a^2\sin^2\theta-48a^2\sin4\theta$;

$dx=6a\cos\theta+3a\cos3\theta$, $dy=-3a\sin3\theta$.

$ds=\sqrt{dx^2+dy^2}=3a(4\cos^2\theta-1)$.

$\therefore s=3a(\theta+\sin2\theta)$.



$$\frac{dy}{dx}=-\frac{\sin 3\theta}{2\cos\theta+\cos 3\theta}=\frac{0}{0},\ \text{when}\ \theta=\frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{5\pi}{3}.$$

$\therefore$ There are cusps at C, D, E, F. When $\theta=\frac{\pi}{6}$ and $\frac{5\pi}{6}$, $x=4a$, $y=0$, $dy/dx=\mp\frac{1}{\sqrt{3}}$; when $\theta=\frac{7\pi}{6}$ and $\frac{11\pi}{6}$, $x=-4a$, $y=0$, $dy/dx=\pm\frac{1}{\sqrt{3}}$.

$\therefore$ A and B are double points. The curve with the tangents at its double points is given in the figure. It is symmetrical to the axes, has four cusps and two double points.

$$(a),\ s=AMB=3a\Big[\theta+\sin 2\theta\Big]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}=3a\left(\frac{\pi}{3}+\sqrt{3}\right)=a(\pi-3\sqrt{3}),$$

$$(b),\ s=AD=3a\Big[\theta+\sin 2\theta\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=\tfrac{1}{2}\pi a,$$

$$(c),\ s=DC=3a\Big[\theta+\sin 2\theta\Big]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}=a(3\sqrt{3}-\pi).$$

Whole length of curve$=2AMB+4AD+2CD=2a(\pi+6\sqrt{3})$.

[The given curve is one of the involutes of a 4-cusped hypocycloid, which could be drawn surrounding the figure. The curve as given by the equations as first proposed is symmetrical to the axes, has two double points but no cusps. EDITOR.]

PROBLEMS.

✗ **47. Proposed by Professor J. SCHEFFER, A. M., Hagerstown, Maryland.**

The floor of a vault forms a square, and all sections parallel to it are squares. The two vertical sections through the middle points of the opposite sides of the floor are equal semi-circles. Find the convex surface and the volume of the vault.

48. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

I have a circular section basin 12 inches in perpendicular height; the diameters are as follows: At base, 2 inches; one inch perpendicular height, 6 inches; two inches perpendicular height, 18 inches; three inches perpendicular height, 54 inches; and so on, the diameter being trebled for every inch in height. After a rain the water in the basin is six inches deep, what was the rainfall?